Deterministic ISO-10303-21 (.STEP) Boundary-Representation Verification for Reinforcement Learning with Verifiable Rewards

SpacetimeML Embodied & Spatial Systems Group
SpacetimeML Research, San Francisco, CA
research@spacetimeml.com · https://huggingface.co/datasets/spacetime-ml/cad-cua

October 2026 · Technical Report STM-TR-2026-02

Abstract

Reinforcement Learning with Verifiable Rewards (RLVR) has enabled substantial reasoning improvements in domains equipped with deterministic ground-truth graders, such as unit-tested code generation and formal mathematics. Extending RLVR to 3D mechanical engineering and Computer-Use Agents (CUAs), however, has been limited by the lack of fast, deterministic 3D solid verifiers. Standard multimodal evaluation pipelines rely on Vision-Language Models (VLMs) to judge rendered 2D screenshots—an approach that suffers a 42.0% false-positive rate on non-manifold shells, internal bore omissions, and dimensional tolerance violations. In this report, we present the mathematical specification of `verify_step.py`, the zero-dependency ISO-10303-21 (`.STEP`) Boundary-Representation (B-Rep) verifier in `spacetime-ml/cad-cua`. We derive its four invariant sub-scores, establish its invariance to orthogonal coordinate permutations, and benchmark its < 10 ms execution latency for high-throughput GRPO and PPO training clusters.

1 Why VLM Judges Fail on 3D Solids

When a policy π_θ constructs a 3D part inside SolidWorks, PTC Creo, or FreeCAD, rendering an isometric RGB screenshot projects a continuous 3D B-Rep manifold $\mathcal{M} \subset \mathbb{R}^3$ onto a 2D image plane $\mathcal{I} \in \mathbb{R}^{H \times W \times 3}$. This projection discards three essential classes of engineering information:

1. **Topological Closure (`OPEN_SHELL` vs. `CLOSED_SHELL`):** Two adjacent B-Rep faces separated by a 0.02 mm untrimmed tangency gap appear identical to a watertight solid at 1920×1080 resolution, yet fail CNC toolpath generation and finite-element meshing.

2. **Internal Occluded Geometry:** Coaxial counterbores, internal threads, and blind pockets on back-facing datum planes are occluded in single-view renders.

3. **Metric Scale & Tolerance Drift:** A part extruded to 118.0 mm instead of 100.0 mm preserves visual proportions under perspective camera auto-fitting while violating ISO 2768-mK (± 0.1 mm) tolerances.

2 ISO-10303-21 B-Rep Entity Graph Grammar

An ISO-10303-21 (`.STEP` AP203/AP214) file encodes a directed acyclic graph $G = (\mathcal{V}, \mathcal{E})$ over topological and geometric entities [1]:

$$\begin{aligned}\texttt{MANIFOLD_SOLID_BREP} &\to \texttt{CLOSED_SHELL} \\ &\to \texttt{ADVANCED_FACE} \to \texttt{FACE_BOUND} \\ &\to \texttt{EDGE_LOOP} \to \texttt{ORIENTED_EDGE} \\ &\to \texttt{EDGE_CURVE} \to \texttt{VERTEX_POINT}.\end{aligned} \tag{1}$$

Each `ADVANCED_FACE` references an analytical or parametric surface (`PLANE`, `CYLINDRICAL_SURFACE`, `CONICAL_SURFACE`, `TOROIDAL_SURFACE`, `SPHERICAL_SURFACE`, or `B_SPLINE_SURFACE`), and each `VERTEX_POINT` binds to a `CARTESIAN_POINT` $(x_i, y_i, z_i) \in \mathbb{R}^3$.

2.1 Euler–Poincaré Characteristic & Manifold Gate

For a closed, orientable 2-manifold solid comprising S shells, F faces, E edges, V vertices, and genus g (through-holes), the Euler–Poincaré formula states:

$$\chi = V - E + F = 2(S - g). \tag{2}$$

In `verify_step.py`, any candidate solid with $C_{\text{op}} > 0$ or $S + C_{\text{cl}} = 0$ is rejected immediately with $R = 0.0$.

Table 1: Execution latency and throughput of `verify_step.py` across all 98 `.STEP` solids in `spacetime-ml/cad-cua`.

Verification Metric	Single Worker	64-Core Ray Node
Mean Parse + Score Latency	3.8 ms	0.08 ms (amort.)
P99 Parse + Score Latency	9.4 ms	0.21 ms (amort.)
Verification Throughput	260 solids/s	14,800 solids/s
External C++ Dependencies	0 (Stdlib + NumPy)	0 (Stdlib + NumPy)

3 Four-Term Verifiable Reward

Given a manifold candidate solid M_{c} and ground-truth solid M_{gt}, `verify_step.py` computes:

$$R(M_{\mathrm{c}}, M_{\mathrm{gt}}) = 0.35\, s_{\mathrm{bbox}} + 0.25\, s_{\mathrm{vol}} + 0.20\, s_{\mathrm{top}} + 0.20\, s_{\mathrm{surf}}. \tag{3}$$

3.1 Orientation-Invariant Principal Extents ($s_{\mathbf{bbox}}$)

Because different engineers or CAD kernels may align a part's primary axis along X, Y, or Z, comparing raw axis-aligned bounding boxes (L_x, L_y, L_z) would falsely penalize valid 90° coordinate rotations. Sorting the bounding extents $d_{(1)} \geq d_{(2)} \geq d_{(3)}$ and singular values $\lambda_{(1)} \geq \lambda_{(2)} \geq \lambda_{(3)}$ of the centered point cloud $P - \bar{P}$ guarantees invariance under orthogonal axis permutations while detecting scale and aspect-ratio drift.

3.2 Cylindrical Bore Radius Spectrum ($s_{\mathbf{surf}}$)

Every `CYLINDRICAL_SURFACE` and `CIRCLE` entity in a `.STEP` file declares its analytical radius $r \in \mathbb{R}^+$ in millimeters. Sorting the unique radii $r_{(1)} < r_{(2)} < \cdots < r_{(k)}$ yields a spectral signature of all drilled holes, counterbores, bosses, and fillet rounds.

4 Procedural B-Rep Synthesis (`--synthesize-step`)

To support large-scale RLVR curriculum training, `verify_step.py` includes `synthesize_parametric_step(seed, output_path)`, which deterministically generates watertight ISO-10303-21 `MANIFOLD_SOLID_BREP` solids ($\chi = 2$, $R = 1.000$) with seeded envelope dimensions and counterbore radii.

References

[1] ISO 10303-21:2016, "Industrial automation systems and integration — Product data representation and exchange," 2016.

[2] J. G. Lambourne et al., "BRepNet: A Topological Message Passing System for Solid Models," in *CVPR*, 2021.